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# Helical magnetic fields via baryon asymmetry

## Campos magnéticos con helicidad vía asimetría bariónica.

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### Abstract

There is strong observational evidence for the presence of large-scale magnetic fields MF in galaxies and clusters, with strength  $\sim \mu G$  and coherence lenght on the order of Kpc. However its origin remains as an outstanding problem. One of the possible explanations is that they have been generated in the early universe. Recently, it has been proposed that helical primordial magnetic fields PMFs, could be generated during the EW or QCD phase transitions, parity-violating processes and predicted by GUT or string theory. Here we concentrate on the study of two mechanisms to generate PMFs, the first one is the  $\nu$ MSM which triggers instability in the Maxwell's equations and leads to the generation of helical PMFs. The second one is the usual electroweak baryogenesis scenario. Finally, we calculate the exact power spectra of these helical PMFs and we show its role in the production of gravitational waves finding a scale-invariant on large scales and an oscillatory motion (damping) for  $k\eta \gg 1$ .

**Keywords:** Primordial magnetic fields, baryon asymmetry of the universe.

### Resumen

Existe una fuerte evidencia observacional de la presencia de campos magnéticos en grandes escalas, tanto en galaxias y cúmulos galácticos con intensidades de  $\sim \mu G$  y con longitudes de coherencia de Kpc. Sin embargo, el origen de estos campos magnéticos es todavía un problema abierto. Una de las explicaciones es que este campo fue generado en etapas tempranas del universo. Recientemente, se ha propuesto campos magnéticos con helicidad, que pudieron ser generados en transiciones de fase electrodébil o de QCD, procesos de violación de paridad y en teorías de gran unificación y teoría de cuerdas. En este trabajo, se estudia dos mecanismos que generan campo magnético con helicidad. El primero es el modelo  $\nu$ MSM activa la inestabilidad en las ecuaciones de Maxwell y conduce a la generación de campos magnéticos con helicidad y el segundo mecanismo bariogenesis electrodébil. Por último, se calcula de una forma exacta

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el espectro de potencias estos campos con helicidad y se muestra la producción de ondas gravitacionales encontrando algunas propiedades en escalas sub y super horizonte.

**Palabras clave:** Campos magnéticos primordiales, asimetría bariónica del universo.

## 1 Electroweak baryogenesis

Before the decoupling epoch the electric conductivity of the plasma  $\sigma$  is very large, therefore we can consider the infinite conductivity limit. In this limit, the induced electric field is zero and the evolution of a primordial magnetic fields is given by  $B(x, \eta) \sim B(x)/a^2(\eta)$ , besides, the magnetic flux and the magnetic helicity are conserved,

$$\frac{d}{d\eta} \int \mathbf{B} \cdot d\mathbf{S} = -\frac{1}{\sigma} \int \nabla \times \nabla \times \mathbf{B} \cdot d\mathbf{S}, \quad (1)$$

$$\frac{d}{d\eta} \mathcal{H}_M = -\frac{1}{\sigma} \int \mathbf{A} \cdot \mathbf{B} dV, \quad (2)$$

where  $A$  is the vector potential and  $\mathcal{H}_M$  is the magnetic helicity which is gauge invariant

$$\mathcal{H}_M = \frac{1}{V} \int \mathbf{A} \cdot \mathbf{B} dV. \quad (3)$$

The magnetic helicity is a very important quantity in astrophysics for different reasons, Grasso & Rubinstein (2001). First,  $\mathcal{H}_M$  coincides with the Chern-Simon number  $N_{CS} \equiv \int \mathbf{A} \cdot \mathbf{B} dV$  which is related to the topological properties of the gauge fields, indeed, a Chern-Simons term different from zero in Maxwell's equations leads to an instability and generation of magnetic fields. The second one, is based on the property from MHD where the presence of helicity can transfer magnetic field power (more efficient) to larger length scales, *inverse cascade*. Therefore we could explain the large magnetic field observed so far. Differents authors have studied generation of PMF via baryon asymmetry, for example in the electroweak context, the violation of the baryon number ( $n_L - n_R$ ) is caused by sphalerons decay and the amount of helical PMF created can be estimated from

$$\frac{d(n_L - n_R)}{dt} \sim -\alpha \frac{d\mathcal{H}_M}{dt} \quad (4)$$

where  $\alpha = 1/137$  is the fine structure constant. Other alternative for generating PMF relies in the interaction of the hypercharge component of the electromagnetic field with the axion by means of the anomaly Grasso & Rubinstein (2001). The Lagrangian for this model can be written as

$$\mathcal{L} \sim -\frac{1}{2} \partial_\mu \theta \partial^\mu \theta - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha \theta F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (5)$$

where  $\theta = \phi_a/f_a$ ,  $\phi_a$  is the axion field,  $f_a$  is the Peccei-Quinn symmetry breaking scale and  $F_{\mu\nu}$  is the electromagnetic field. Grasso & Rubinstein (2001) shows that in

presence of QCD sphaleron the axion equation of motion is

$$\ddot{\phi} + (3H + \frac{\alpha^4 T^3}{f_a^2})\dot{\phi} = 0. \quad (6)$$

Using the later equation, Grasso & Rubinstein (2001) estimated that in the temperature range  $1\text{GeV} > T > 10\text{MeV}$  and  $f_a > 109 \text{ GeV}$ , the existence of a small PMF is possible (with the QCD sphaleron out of thermal equilibrium) of the order of  $B \cdot A \sim 10^{-22}$ .

## 2 The $\nu\text{MSM}$

The Standard Model answers many of the questions about the structure and stability of matter, but it's not complete, there are still many unanswered questions like neutrino oscillation, matter-antimatter asymmetry and dark matter among others, de Medeiros Varzielas et al. (2012). The model  $\nu\text{MSM}$  was introduced to overcome some of these problems. In this model, the tiny values of the neutrino masses are related to the small Yukawa Coupling constants between sterile neutrinos and left-handed leptonic doublets. In this model, three singlets ( $N_R$ ) are introduced, these mix with the standard neutrinos and we get the following mass Lagrangian

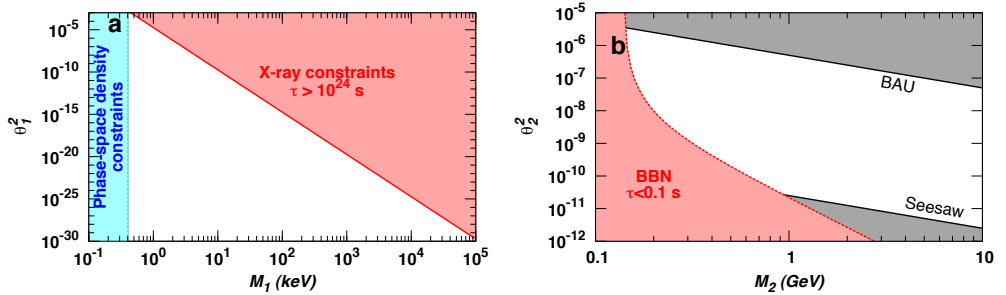
$$-\mathcal{L}_{\text{mass}} = \frac{1}{2} (\overline{(v_L)^c} \quad \overline{N_R}) \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} v_L \\ (N_R)^c \end{pmatrix} + \text{h.c.}, \quad (7)$$

where  $v_L$  are the standard neutrinos,  $m_D$  are the Dirac-mass matrix and  $M_R$  are the Majorana-mass matrix. When this matrix is diagonalized, we get the following physics states

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} V_{PMNS}^\dagger v - V_{PMNS}^\dagger B_1 N^c \\ B_1^\dagger v + N^c \end{pmatrix}_L + \begin{pmatrix} V_{PMNS}^T v^c - V_{PMNS}^T B_1^* N \\ B_1^T v^c + N \end{pmatrix}_R, \quad (8)$$

where  $V_{PMNS}$  is the Pontecorvo-Maki-Nakagawa-Sakata matrix and  $B_1^\dagger = M_R^{-1} m_D$  is a seesaw-factor.  $\xi_1$  are the light neutrinos corresponding to the standard model neutrinos, while  $\xi_2$  are the heavy neutrinos. One of these heavy neutrinos ( $N_1$ ) will be chosen like dark matter candidate while the other two ones ( $N_2, N_3$ ) will generate the oscillation neutrino patterns and will generate baryon asymmetry. This choice generates conditions between the values of its mass and mixing angles. In particular, if we consider the masses of the light neutrinos on the order of mass particles from the standard model, the interaction should be *superweak*. The constraint of these free parameters, which comes from cosmology, astrophysics and neutrino oscillation, are shown in the figure 1.

An important characteristic of this model, is that baryon asymmetry could be possible via leptogenesis. Indeed, if the three Sakharov conditions are satisfied Sakharov (2009), the initial lepton asymmetry generated by violating CP in neutrino oscillation, is converted to baryon asymmetry through electroweak anomaly (with field configurations no-conserved baryon number -*sphalerons*-). In this way, the helical PMF is produced as we see in equation (4).



**Figure 1:** Constrains of the neutrinos mass and mixing angles for  $N_1$  (left) and  $N_{2,3}$  (right), Boyarsky et al. (2009).

### The statistics for helical PMFs

We consider a stochastic PMF generated before recombination. The power spectrum which is defined as the Fourier transform of the two point correlation can be written as

$$\langle B_j(\mathbf{k}) B_l^*(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') [P_{jl} S(k) + i \epsilon_{jlm} \hat{k}_m A(k)], \quad (9)$$

here  $S(k)$  and  $A(k)$  are the symmetric and helical part of the PMF power spectrum respectively,  $P_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j$  is the transverse plane projector and  $\epsilon_{ijl}$  is the antisymmetric tensor. We scale the PMF by a power law (for  $k < k_D$ )

$$S(k) = \frac{B_\lambda^2 (2\pi)^2 \lambda^{n_S+3}}{\Gamma\left(\frac{n_S+3}{2}\right)} k^{n_S}, \quad A(k) = \frac{B_\lambda^2 (2\pi)^2 \lambda^{n_A+3}}{\Gamma\left(\frac{n_A+4}{2}\right)} k^{n_A}, \quad (10)$$

where  $n_S, n_A$ , are the spectral indices for symmetric and helical parts respectively and  $k_D$  is an ultraviolet cut-off (for a dependence of an infrared cutoff and its dependence with spectral index, see Hortua & Castañeda (2014)).

The anisotropic trace-free part written in Fourier space is given by

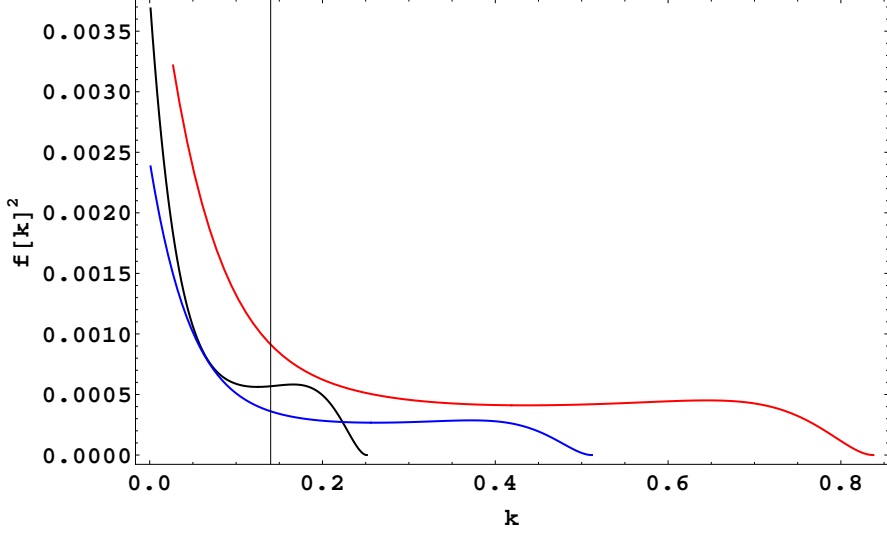
$$\Pi_{ij} = \int \frac{d^3 k'}{2(2\pi)^4} \left[ B_i(k') B_j(k - k') - \frac{\delta_{ij}}{3} B_l(k') B^l(k - k') \right],$$

and working with the linear cosmological perturbation theory, one can find that anisotropic tensor is source of gravitational waves (GWs) Caprini et al. (2004)

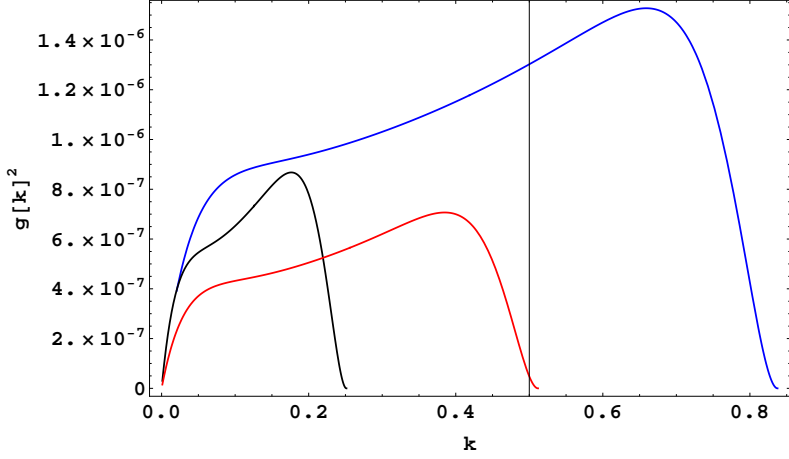
$$\ddot{h} + 2H\dot{h} + k^2 h \sim \Pi, \quad (11)$$

where  $h$  is the tensor part in the metric associated with GWs and  $H$  the Hubble parameter. Now, we define the two point correlation function for the anisotropic trace-free part as

$$\langle \Pi_{ij}^{(T)}(k) \Pi_{ij}^{(T)*}(k') \rangle = 4(2\pi)^3 (f^2(k) + 2ig^2(k)) \delta^3(k - k'), \quad (12)$$



**Figure 2:** The symmetric power spectra  $f^2(k)$  for  $n_S = 8/, n_A = 9$ ,  $n_S = 4/, n_A = 5$  and  $n_S = 6/, n_A = 7$



**Figure 3:** The helical power spectra  $g^2(k)$  for  $n_S = 6/, n_A = 7$ ,  $n_S = 4/, n_A = 5$  and  $n_S = 8/, n_A = 9$

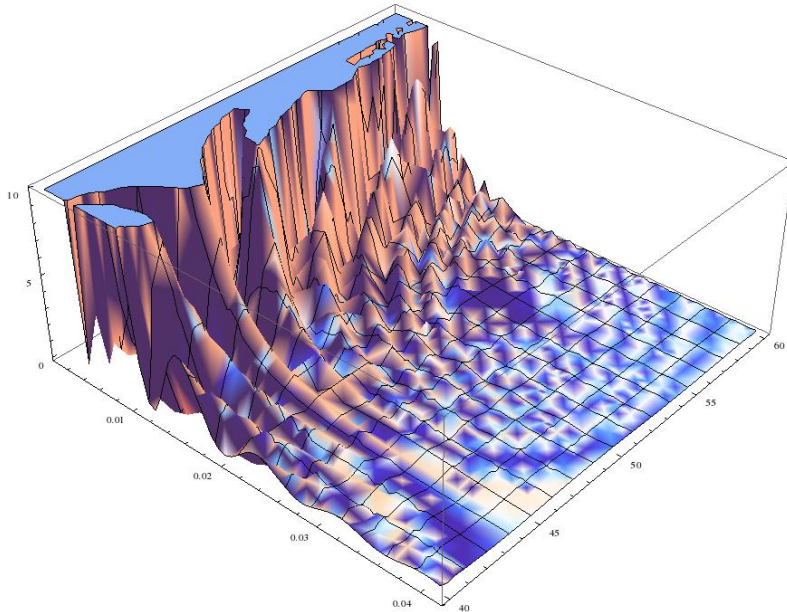
hence, using the Wick theorem and the later equations one can arrives to

$$g^2(k) = \frac{8}{512\pi^5} \int d^3k' [\beta(1 + \gamma^2)] S(k') A(|\mathbf{k} - \mathbf{k}'|), \quad (13)$$

$$f^2(k) = \frac{1}{512\pi^5} \int d^3k' [(1 + 2\gamma^2 + \gamma^2\beta^2) S(k') S(|\mathbf{k} - \mathbf{k}'|) + 4A(k') A(|\mathbf{k} - \mathbf{k}'|)] \quad (14)$$

with  $\beta = \frac{\mathbf{k} \cdot (\mathbf{k} - \mathbf{k}')}{k|\mathbf{k} - \mathbf{k}'|}$ ,  $\gamma = \frac{\mathbf{k} \cdot \mathbf{k}'}{kk'}$ , which is in agreement with Caprini et al. (2004). The power spectra for symmetric and helical part are shown in figures 2 and 3.

For studying the behavior of GW induced by PMFs, we use the equation (11), obtaining the figure 4. Here we can see the behavior of GW spectra in the interest scale.



**Figure 4:** The gravity wave power spectra produced by PMFs.

Basically, we have found that GWs spectra is constant in super-horizon scale (type Harrison-Zeldovich spectrum) whilst at small scales, the spectra oscillates rapidly and decay within time.

### 3 Discussion

In this paper we have shown the exact solution of power spectra generated by helical primordial magnetic fields created after inflation (causal) for some spectral indices. These fields could be originated by processes in the early universe. More specifically, we studied some mechanisms which generate magnetic fields via baryon asymmetry of the universe. These fields are interesting in sense that creation of magnetic fields comes accompanied by helicity, therefore, the dynamo mechanisms are more efficient and could drive to the explanation of large magnetic fields observed today.

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